Journal of Engineering Physics and Thermophysics, Vol. 75, No. 5, 2002

COMPARATIVE EVALUATION OF THE RESISTANCE OF POLYETHYLENE AND STEEL PIPELINES TO A FROST CRACK

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UDC 678.067.5.017

We have carried out a comparative evaluation of the resistance of polyethylene and steel pipes to the action of a frost crack, i.e., to the discontinuity in the near-surface ground layer resulting from the occurrence of complex geocryological processes in the season-thawing layer of ever-frozen grounds.

Let us consider a model situation in which the process of interaction between the frost crack and the pipeline is presented as the propagation of a macroscopic cross crack in a plate where a cylindrical tube is installed (see Fig. 1a). The length of the cylindrical crack of scaling around the pipe L_0 on the cross-macrocrack front can be determined following the approach of [1].

Assuming that the characteristic dimensions L_0 and r are small compared to those of the macrocrack (here the pipe is treated as a solid "thread"), we consider that the nonzero distribution of the shear stress over the extension of the scaling crack along the side surface of the pipe lies entirely in the region of action of the asymptotics of the macrocrack front. Consequently, the pulling force that acts upon the pipe on the crack front can be found from the averaged asymptotics of the stress σ_z (for the equivalent orthotropic body simulating the properties of the ground) near the crack front. Using the known asymptotics on the crack front in the anisotropic body [1], we find

$$\sigma_z = \frac{AK_{\mathrm{I}}}{\sqrt{2\pi z}}\,, \quad A = \frac{1}{2}\,\mathrm{Re}\,\frac{1+i}{\sqrt{\mu}}\,,$$

$$\mu = \sqrt{\nu_x - \frac{E_x}{2G} + i\sqrt{\frac{E_x}{E_z} - \left(\nu_x - \frac{E_x}{2G}\right)^2}}\ . \tag{1}$$

The equation of force equilibrium for the half of the cylinder must hold on the macrocrack front:

$$\pi r^2 \sigma_z = 2\pi r \tau_{\mathrm{f}} L_0 + 2\pi r \int_{L_0}^{\infty} \tau_{rz}\, dz\ . \tag{2}$$

Transformation of Eq. (2) with account for Eq. (1) yields

$$\frac{ArK_{\mathrm{I}}}{2\sqrt{2\pi L_0}} = \tau_{\mathrm{f}} L_0 + \int_{L_0}^{\infty} \tau_{rz}\, dz\ . \tag{3}$$

Following [1], we approximate the stress τ_{rz} on the cylinder boundary by the expression

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1062-0125/02/7505-1054$27.00

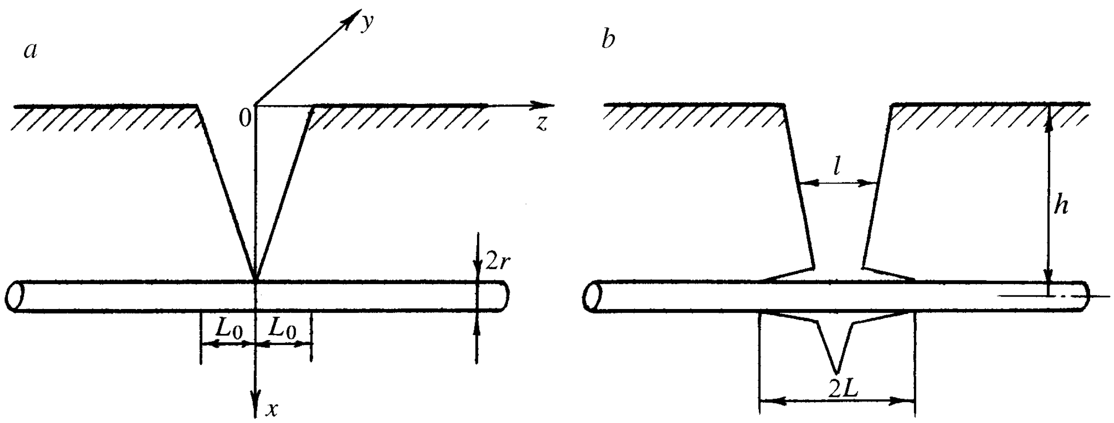

Fig. 1. Model scheme of interaction between the frost crack and the gas pipe.

$$\tau_{rz} = \begin{cases} \tau_f , & |z| < L_0 , \\ \dfrac{K_{II} L_0 \sqrt{3L_0}}{\sqrt{\pi (z^6 - L_0^6)}} , & |z| > L_0 . \end{cases} \tag{4}$$

In our case, the stress intensity factor K_{II} is equal to K_{IIcr} — to the critical value determining the breaking moment for the adhesive joint pipe surface–ground, i.e., to the onset of the scaling process. We confine ourselves to consideration of the simplest case, setting $\tau_f = 0$. Using Eqs. (3) and (4), we find

$$L_0 = \frac{ArK_I}{BK_{IIc}} . \tag{5}$$

Thus, the length of the symmetric cylindrical crack of scaling ($2L_0$) is proportional to K_I at the tip of the macrocrack (of the frost crack) and inversely proportional to K_{IIcr}, i.e., to the critical stress intensity factor of shear stresses for the scaling crack.

The process of interaction between the frost crack and the pipeline involves two stages: 1) the frost crack reaches the gas-pipe boundary with the subsequent formation of a cylindrical scaling crack of length $2L_0$ (see Fig. 1a); 2) upon reaching the value of $K_I = K_{Icr}$, a macrocrack develops on the "dark" side of the pipe and continues its propagation into the depth of the ground massif. With a rather large store of elastic energy in the massif, the processes of scaling and development of the macrocrack into the depth (below the level of laying of the pipeline) will be simultaneous, in practice. When the macrocrack stops, its edges will take a certain configuration (see Fig. 1b); in particular, at the gas-pipe depth h the frost crack will have a certain opening $l(h) = 2L - 2L_0$. Then as the criterion of integrity conservation of the pipe we can use the inequality

$$\varepsilon = \frac{2L - 2L_0}{2L_0} < \varepsilon_{br} , \tag{6}$$

where $\varepsilon_{br} = \varepsilon_{br}(T, v)$.

To carry out a quantitative comparative evaluation of the resistance of the steel and plastic gas pipes to the frost crack according to relations (5) and (6), it is necessary to know the numerical or relative values of the characteristics for the indicated materials K_{IIcr} and ε_{br}.

No direct measurements of the quantity K_{IIcr} have been carried out. We use the experimental data of [2], which indicate that the resistance to the shear (scaling) of the ground and ice over the surface of the plastics is 2.5–5 times less than over the steel specimens. The experiments were conducted on a Sadovskii plane-shear device. The limiting value of the shear resistance for frozen grounds and ice was determined at T = 268–267 K from the formula

$$\tau_{cr} = \frac{P_{cr}}{F} . \tag{7}$$

In particular, for the specimens of the polyethylene pipes we obtained the following results: τ_{cr} = 0.3–0.35 MPa for the loams, τ_{cr} = 0.15 MPa for the ice, and τ_{cr} = 0.15–0.2 MPa for the sand.

Assuming that $K_{IIcr} \sim \tau_{cr}$, we evaluate the scaling length L_0 for the steel and polyethylene pipes: τ_{cr}^{st} = (2.5–5)τ_{cr}^{pe} and select for evaluation $\tau_{cr}^{st} = 2.5\tau_{cr}^{pe}$ with a "margin"; then

$$K_{IIcr}^{st} = 2.5K_{IIcr}^{pe} , \tag{8}$$

$$L_0^{st} = \frac{Ar_0K_I}{B{\cdot}2.5K_{IIcr}^{pe}} = C\frac{1}{2.5K_{IIcr}^{pe}} , \quad C - \text{const}, \tag{9}$$

$$L_0^{pe} = \frac{Ar_0K_I}{BK_{IIcr}^{pe}} = C\frac{1}{K_{IIcr}^{pe}} , \tag{10}$$

$$\varepsilon^{st} = \frac{2L^{st} - 2L_0^{st}}{2L_0^{st}} = l^{st}\frac{2.5K_{IIcr}^{pe}}{2C} < \varepsilon_{br}^{st} , \tag{11}$$

$$\varepsilon^{pe} = \frac{2L^{pe} - 2L_0^{pe}}{2L_0^{pe}} = l^{pe}\frac{K_{IIcr}^{pe}}{2C} < \varepsilon_{br}^{pe} . \tag{12}$$

We assume that ε_{br}^{st} = 18%. This value has been taken from the literature data [3] obtained at a test temperature of about 293 K and an extension rate of the specimen of 2 mm/min. For the steel we will use the given quantity $\varepsilon_{br}^{st} = 18{\cdot}10^{-2}$, neglecting its reduction with decrease in the temperature and increase in the deformation rate, since we are not aware of the extension rate of the frost crack.

The resistance of the polyethylene pipe to the frost crack will be evaluated by using the temperature-time similarity principle [4]. The greatest possible loading rate of the polyethylene pipe is the velocity of sound in the frozen ground, which can be taken to be equal to that in ice ($3{\cdot}10^3$ m/sec). This velocity is $3{\cdot}10^6$ times higher than the loading rate of material in standard experiments for tension, which are carried out at a rate of $3{\cdot}10^{-3}$ m/sec.

Assuming [5] that the main contribution to the deformation is made by one relaxation mechanism, it can be considered in the first approximation that the characteristic time varies with temperature exponentially:

$$t = t_0 \exp\left(\frac{U}{kT}\right). \tag{13}$$

The value of the activation energy U is of the order of 1 eV, while the quantity kT at room temperature is about 0.025 eV; thus, every 7 to 8 K, the deformation rate changes exp $\approx$ 2.718 times.

Consequently, a variation of 10^6 in the rate is equivalent to a variation of approximately 90–100 K in the temperature. Assuming that the appearance of the frost crack is a consequence of a temperature gradient in the depth of freezing, we set T_0 = 273 K and $v_0 = 3{\cdot}10^{-6}$ m/sec, while the corresponding equivalent loading condition has the parameters T = 183 K and $v = 3{\cdot}10^{-3}$ m/sec.

According to the existing data on the deformation-strength characteristics for polyethylene in a wide temperature interval (333–153 K), the limit relative breaking deformation for polyethylene is ε_{fr}^{pe} (183 K and $3{\cdot}10^{-3}$ m/sec) $\approx 50{\cdot}10^{-2}$ [4, 6].

Then, using Eqs. (11) and (12), for the critical maximum admissible values of the opening of the frost cracks at the gas-pipe depth $l(h) = 2L - 2L_0 = l_{cr}$ we obtain

$$l_{cr}^{st} = \frac{2C}{2.5K_{IIcr}^{pe}} \varepsilon_{br}^{st} \approx 0.8 \cdot 0.18 \frac{C}{K_{IIcr}^{pe}} \approx 0.15 \frac{C}{K_{IIcr}^{pe}}, \quad l_{cr}^{pe} = \frac{2C}{K_{IIcr}^{pe}} \varepsilon_{br}^{pe} \approx 2 \cdot 0.5 \frac{C}{K_{IIcr}^{pe}} \approx \frac{C}{K_{IIcr}^{pe}}, \tag{14}$$

whence $l_{cr}^{pe} \approx 7 l_{cr}^{st}$; if $\tau_{cr}^{st} = 5\tau_{cr}^{pe}$, then the estimate is doubled:

$$l_{cr}^{pe} \approx (7 - 14)\, l_{cr}^{st}. \tag{15}$$

Thus, the polyethylene gas pipe is capable of conserving its normal operation relative to the action of a frost crack that is 7–14 times more powerful in the criterion of opening of the crack than the steel pipe.

The above evaluation can be carried out in a rougher approximation. By convention we assume that the forces in the pipe do not affect the width of the crack opening $l(h)$. We assume that when the crack reaches the pipe surface, scaling of the pipe from the ground on the portion $2L_0 = mh$ occurs; then the crack propagates into the depth of the ground massif. Considering that the coefficients m^{st} and m^{pe} are proportional to the adhesive characteristics of the steel–ground and polyethylene–ground interaction and using the values of ε_{br}^{st} and ε_{br}^{pe}, we obtain

$$\varepsilon_{br}^{st} = \frac{l_{cr}^{st}}{m^{st} h}, \quad \varepsilon_{br}^{pe} = \frac{l_{cr}^{pe}}{m^{pe} h} = \frac{l_{cr}^{pe}}{m^{st} \cdot 2.5h},$$

from which

$$l_{cr}^{pe} = \frac{\varepsilon_{br}^{pe}}{\varepsilon_{br}^{st}} 2.5\, l_{cr}^{st} \approx 7 l_{cr}^{st},$$

which coincides with estimate (15).

NOTATION

L_0, half-length of the cylindrical scaling crack (see Fig. 1a), m; r, pipe radius, m; σ_z, stresses on the crack front, MPa; K_I, factor of separation stress intensity on the macrocrack front, MPa·m$^{1/2}$; i, complex unit ($\sqrt{-1}$); E_x, E_z, and G, elastic moduli along the x and z axes and the shear elastic modulus in the xz plane, respectively, MPa; ν_x, Poisson coefficient along the x axis; τ_f, force of friction of the pipe on the ground per unit area of contact, MPa; τ_{rz}, shear stresses on the boundary of the pipe–ground contact, MPa; K_{II}, factor of shear-stress intensity, MPa·m$^{1/2}$; B, a certain constant number determined experimentally or from accurate calculation [1]; K_{IIcr}, critical factor of shear-stress intensity for the scaling crack, MPa·m$^{1/2}$; F, P_{cr}, and τ_{cr}, area of the specimen–ground (ice) contact and critical values of the loading and the scaling stress at which the separation of the ground or ice from the specimen surface was recorded in the experiment performed on the Sadovskii device, m^2, N, and MPa; U, activation energy of the "main" mechanism of deformation, eV; k, Boltzmann constant, eV/K; t, time, sec; t_0, time constant, sec; h, gas-pipe depth, m; ε, relative tensile deformation; ε_{br}, limiting relative breaking deformation; T, temperature, K; v, deformation rate of the pipe material, sec^{-1}; L_0^{st} and L_0^{pe}, half-length of the cylindrical scaling crack for the steel and polyethylene pipes, respectively, m; l, opening of the crack in the ground, i.e., distance between the edges of the frost crack, m; L, half-length of the portion of the pipe scaled from the ground and extended by the edges of the frost pipe (see Fig. 1b), m; ε_{br}^{st} and ε_{br}^{pe}, limit relative deformation in breaking of the steel and polyethylene pipes, respectively; m, coefficient; l_{cr}^{st} and l_{cr}^{pe}, critical opening of the frost crack for the steel and polyethylene pipes, respectively, at the pipeline depth, m. Superscripts and subscripts: cr, critical; br, breaking; f, friction; st and pe, steel and polyethylene pipes, respectively.

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